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جامعة بيرزيت
BIRZEIT UNIVERSITY
Mathematics Department
Math 331
Final Exam /2006/07- 1st.Semester

[redacted]

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Part One (choose the most correct answer)

(1) Find m such that y^m is an integrating factor of the ODE:

$$-2xy + (2x^2 + y)\frac{dy}{dx} = 0$$

(a) -2
(b) 2
(c) ~~3~~
$\boxed{\text{(d) -3}}$
(e) None of the above.

$4x$
$-2x$
$-2xy\,dx + (2x^2 y)\,dy = 0$
$M \quad N$

$Q(y) = \frac{4x + 2x}{-2xy} = \frac{6x}{-2xy} = \frac{-3}{y}$

$M(y) = e^{-\int \frac{3}{y} dy} = e^{-3\ln y} = y^{-3}$

(2) Indicate the type of the following differential equation:

$$\sqrt{x+y} + \frac{dy}{dx} = 0$$

(a) Exact
(b) Linear
(c) Separable
(d) Homogeneous
(e) None of the above

Table of Laplace Transforms

	$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$		$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	e^{at}	$\frac{1}{s-a}$
3.	$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\ n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n e^{at},\ n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{e^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	e^{-cs}
27.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$	28.	$u_c(t)g(t)$	$e^{-cs}\mathcal{L}\{g(t+c)\}$
29.	$e^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$

37. $f^{(n)}(t)$ $\quad s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$

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QUESTION THREE: [18 points]

Consider the system:

$$\frac{dx}{dt} = 3x - y + \sin t$$

$$\frac{dy}{dt} = -x + 3y + \cos t$$

(a) Write the system in matrix form $\dfrac{dX}{dt} = AX + B(t)$.

(b) Find the general solution of the homogeneous system:

$$\frac{dx}{dt} = 3x - y$$

$$\frac{dy}{dt} = -x + 3y$$

(c) Find by the method of undetermined coefficient a particular solution of the non-homogeneous system.

QUESTION TWO: [15 points]

Find the solution to the initial value problem:

$$t^2 \frac{d^2 y}{dt^2} - 2y = t^2, \quad y(1) = 3, \quad y'(1) = -1.$$

14. Given the initial value problem:

$$y''+xy'-2y=0, \quad y(1)=1, \quad y'(1)=2.$$

The problem has a unique solution of the form $y=\sum_{n=1}^{\infty} a_n (x-1)^n$ valid for

$|x-1|<R$ where a_0, a_1 and R are given by:

(a) $a_0=a_1=1; \quad R=\infty$

(b) $a_0=a_1=1; \quad R=1$

(c) $a_0=1, a_1=2; \quad R=\infty$

(d) $a_0=1, a_1=2; \quad R=1$

15. Given the initial value problem:

$$y''+xy'-2y=0, \quad y(1)=1, \quad y'(1)=2.$$

The problem has a unique solution of the form $y=\sum_{n=1}^{\infty} a_n (x-1)^n$ valid for

$|x-1|<R$ with coefficients $a_2, \quad a_3$ given by:

(a) $a_2=1, \quad a_3=0$

(b) $a_2=\frac{1}{2}, \quad a_3=\frac{1}{6}$

(c) $a_2=0, \quad a_3=\frac{1}{3}$

(d) $a_2=1, \quad a_3=\frac{1}{6}$

16. Solve the initial value problem

$$\frac{d^2y}{dt^2}\frac{dy}{dt}=3, \quad y(1)=1, \quad y'(1)=1.$$

Then compute $y(5)$.

(a) 117/9 (b) 125/9 (c) 133/9 (d) 107/9 .

17. The differential equation

$$(12y^2+9xy)+(18xy+6x^2)\frac{dy}{dx}=0$$

has an integrating factor $\mu=x^p y^q$. Find p and q.

(a) $p=1, q=2$

(b) $p=2, q=2$

(c) $p=1, q=1$

(d) $p=2, q=1$

18. Given the differential equation:

$$t^2y''-ty'+y=0. \qquad (*)$$

Use The Abel's theorem to find the Wronskian $W(y_1,y_2)$ of tow linearly independent solutions to (*). Given that $y_1(t)=t$ is a solution to (*) find a second linearly independent solution $y_2(t)$:

(a) $W=Ct, \quad y_2(t)=t\ln t$

(b) $W=Ct, \quad y_2(t)=t^2/2$

(c) $W=Ct^2, \quad y_2(t)=t\ln t$

(d) $W=t^2, \quad y_2(t)=t^2$.

9. The Laplace transform of $f(t) = t\sinh(2t)$ is

(a) $\frac{2}{s^2-4}$

(b) $\frac{4s}{(s^2-4)^2}$

(c) $\frac{-4s}{(s^2-4)^2}$

(d) $\frac{2}{(s^2-4)^2}$

10. Given the Laplace transform

$$\mathcal{L}\{\sin(\sqrt{t})\} = \frac{\sqrt{\pi}}{2} e^{-1/(4s)} s^{-3/2}.$$

Find the Laplace transform $\mathcal{L}\{e^t \sin(\sqrt{t})\}$.

(a) $\frac{\sqrt{\pi}}{2} e^{\frac{1}{4(s+1)}} (s+1)^{-3/2}$

(b) $\frac{\sqrt{\pi}}{2} e^{\frac{-1}{4(s-1)}} (s-1)^{-3/2}$

(c) $\frac{\sqrt{\pi}}{2} e^{\frac{-s-1}{4s}} s^{-3/2}$

(d) $\frac{\sqrt{\pi}}{2} e^{\frac{s-1}{4s}} s^{-3/2}$

11. Find the Laplace transform of

$$f(t) = \begin{cases} 0 & \text{if } t \le 0 \\ t & \text{if } 0 < t \le 1 \\ 2-t & \text{if } 1 < t \le 2 \\ 0 & \text{if } t > 2 \end{cases}$$

(a) $\left(\frac{1-e^{-s}}{s}\right)^2$

(b) $\left(\frac{1+e^{-s}}{s}\right)^2$

(c) $\frac{1-e^{-s}}{s}$

(d) $\frac{1+e^{-s}}{s}$

12. Compute the convolution product $\sin t * \cos(2t)$

(a) $\sin t - \sin(2t)$

(b) $\frac{1}{3}(\cos t - \cos(2t))$.

(c) $\sin t - \cos(2t)$

(d) $\cos t - \sin(2t)$

13. The minimum radius of convergence of the power series solution of the ODE $(x^2 - 2x + 2)y'' + xy' + \frac{1}{x+2} y = 0$ about $x_0 = 0$ is:

(a) 1

(b) 2

(c) $\sqrt{2}$

(d) ∞.

4. Find by the method of undetermined coefficient a suitable form to find a particular solution of the differential equation:

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5=e^{-2x}\cos x$$

(a) $y=Ax^2e^{-2x}\cos x$

(b) $y=Ae^{-2x}\cos x+Be^{-2x}\sin x$

(c) $y=Axe^{-2x}\cos x+Bxe^{-2x}\sin x$

(d) None of the above

5. A cup of coffee initially at temperature $T_0=172^0$F is brought into a room at temperature $S=72^0$F, which can be regarded as constant. After 6 minutes the temperature of the coffee is 142^0F. What temperature will the coffee be an additional 6 minutes later? Choose the closest value from the list below.

(a) 120^0F (b) 110^0F (c) 140^0F (d) 130^0F

6. Find the general solution of the ordinary differential equation

$$\frac{d^3y}{dt^3}-3\frac{d^2y}{dt^2}-4\frac{dy}{dt}+12y=0.$$

(a) $y=C_1e^t+C_2te^t+C_3t^2e^t$ (b) $y=C_1e^{3t}+C_2e^{2t}+C_3te^{2t}$

(c) $y=C_1e^{3t}+C_2e^{2t}+C_3e^{-2t}$ (d) $y=C_1e^{-3t}+C_2e^{-2t}+C_3e^{2t}$

7. Find the orthogonal trajectories to the 1-parameter family of parabolas (curves perpendicular to the original family)

$$2x+\alpha=y^2$$

Here α is a parameter and in the proposed solutions β is the parameter.

(a) $2y+\beta=x^2$ (b) $y=\beta e^{-x}$

(c) $y=e^{\beta x}$. (d) $y=e^{-\beta x}$.

8. Find the inverse Laplace transform of : $\frac{2s+3}{s(s^2-1)}$

(a) $-3+\frac{5}{2}e^t+\frac{1}{2}e^{-t}$ (b) $3-3\cos t+2\sin t$

(c) $\frac{5}{2}e^t+\frac{1}{2}e^{-t}$ (d) $3\cos t+2\sin t$

QUESTION ONE: (MULTIPLE CHOICE) [72 POINTS]

Question	Your answer	Question	Your answer
1	b	10	b
2	d	11	a
3	a	12	b
4	c	13	c
5	a	14	c
6	c	15	c
7	d	16	c
8	a	17	c
9	b	18	a

CHOOSE the right answer and then fill the table above:

1. The differential equation $2x(2y-1)dx+(2x^2+2)dy=0$ is

(a) only exact (b) both exact and separable

(c) only separable (d) neither exact nor separable.

2. Solve the initial value problem

$$\frac{dy}{dt}=(1+y^2)t,\quad y(0)=1.$$

(a) $\ln(y^2+1)=\frac{t^2}{2}$

(b) $y(t)=\tan(t^2$ [illegible]$)+1$

(c) $y(t)=\tan(t)+1$

(d) $y(t)=\tan\left(\frac{t^2}{2}+\frac{\pi}{4}\right)$

3. Find the general solution of the differential equation

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+13y=0$$

(a) $y=Ae^{2x}\cos(3x)+Be^{2x}\sin(3x)$

(b) $y=Ae^{2x}+Bxe^{2x}$

(c) $y=Ae^{-x}+Be^{5x}$

(d) None of the above.

Final Exam 1st Semester/2005-06

Student name: [redacted] Student no.: 1041323

$1.2\times4 = 4.8$

40

$c(0) = 2g/L$

$Q(0) = 40\times2 = 80g$

(SHOW YOUR WORK)

Q#1 (5 points each)

(1) A very large tank contains 40 L brine of concentration 2g/L salt. Brine of concentration 1.2 g/L salt flows into the tank at 4L/min and the well-mixed solution is pumped out at 2 L/min. Assuming that the tank does not overflow what is the concentration of salt in the tank after 10 min? (Choose the closest value.)

(a) 1.555 g/L (b) 1.448 g/L (c) 1.327 g/L

(d) 1.298 g/L (e) 1.000 g/L

$\frac{dQ}{dt} = 4.8 - \frac{2Q}{40+2t}$ linear D.E

$\frac{dQ}{dt} + \frac{Q}{20+t} = 4.8$

$\mu: e^{\int \frac{1}{20+t}} = e^{\ln(20+t)} = 20+t$

$Q = \frac{1}{(20+t)}\int (20+t)4.8\,dt + \frac{c}{20+t}$

$= \frac{1}{20+t}\int 96 + 4.8t\,dt$

$= \frac{96t}{20+t} + \frac{2.4t^2}{20+t} + \frac{c}{20+t}$

$Q = \frac{96t + 2.4t^2 + c}{20+t}$

$Q(0) = 80 \Rightarrow \frac{c}{20} = 80 \quad c = 1600$

concentration $= \frac{Q}{40+2t} = \frac{96t + 2.4t^2 + 1600}{2(20+t)^2} = \frac{960+240+1600}{2\times900} = 1.555 g/L$

(2) Consider the differential equation

(*) $$(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \frac{y}{x+2} = 0$$

What would you say about the series solution of (*):

(a) The differential equation has a solution of the form $y = \sum_{n=0}^{\infty} a_n (x-1)^n$.

(b) The differential equation has a solution of the form $y = \sum_{n=0}^{\infty} a_n (x-1)^{n+r}$.

(c) The differential equation has a solution of the form $y = \sum_{n=0}^{\infty} a_n x^n$.

(d) Both (a) and (c).

(e) Both (b) and (c).

$p(x) = x^2 - 1 \Rightarrow |x| < 1$

$x=0$ is ordinary pt. $\Rightarrow y = \sum a_n x^n$

$x = 1, -1$ regular singular pt.

for $x=1$: $\lim_{x\to1} (x-1)\frac{x}{x^2-1} = \lim \frac{x}{1+x} = \frac{1}{2}$ regular singular

$\lim_{x\to1} (x-1)^2 \frac{x}{x^2}... = 0$

$20 + 18 \times 4 = 92$

FINAL

BIRZEIT UNIVERSITY

Mathematics Department

Math 331

Final Exam /2004

Student no.: [redacted]

Student name: [redacted]

Instructer : Wasem AL-Kaibi

Q#1 (20 Points) Classify the following ordinary differential equations using the choice at the right. Each type of equation occurs one in the list. (Hint: read the list of choices first.)

D	$xy' = y^2 + 1$	A.	Exact 1st order
A	$2xy - y^2 + (x^2 - 2xy)y' = 0$	B.	1st order homogeneous equation which is not exact
B	$(x^2 + y^2)y' = (5xy - y^2)$	C.	Linear 1st order
H	$y'' - 6y' + 6y = \sin x$	D.	Separable 1st order
F	$y' + xy = x^2y^2$	E.	2nd order Euler equation
J	$x^2y'' + (x^2 + x)y' - y = 0$	F.	Bernoulli equation
E	$x^2y'' - 32xy' + 50y = 0$	G.	Homogeneous 2nd order constant coefficient linear
G	$y'' + 5y' - 7y = 0$	H.	Inhomogeneous 2nd order constant coefficient linear
I	$y'' + xy' - 3y = 0$	I.	2nd order linear, neither Euler nor constant coefficient, with no singular points
K	$5y''' - 6y'' + y' - y = 0$	J.	2nd order linear, neither Euler not constant coefficient, with a regular singular point at $x_0 = 0$
C	$y' + e^x y = \cos x$	K.	Constant coefficient 3rd order linear

$(x^2+y^2)\frac{dy}{dx} + (y^2 - 5xy) = 0$

$(x^2+y^2)\,dy + (y^2 - 5xy)\,dx = 0$

$xy' = y^2 + 1$

$x\frac{dy}{dx} = y^2 + 1$

$\frac{dx}{x} = dy\,(y^2+1)$

$\frac{(x^2+x)}{x}(x+1) = 1$

0

(3) Consider the differential equation

$$(*) \qquad (x^2-1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\frac{y}{x+2}=0$$

What would you say about the series solution of (*):

(a) The differential equation has a solution of the form $y=\sum_{n=0}^{\infty}a_n(x-1)^n$.

(b) The differential equation has a solution of the form $y=\sum_{n=0}^{\infty}a_n(x-1)^{n+r}$.

(c) The differential equation has a solution of the form $y=\sum_{n=0}^{\infty}a_n x^n$.

(d) Both (a) and (c).

(e) Both (b) and (c).

$4x$ $\quad 4x$

(4)The differential equation $2x(2y-1)dx+(2x^2+2)dy=0$ is

$(2x^2+2)dy=-2x(2y-1)$

$\frac{dy}{2y-1}=\frac{-2x}{2x^2+2}$

(a) only exact

(b) only separable

(c) both exact and separable

(d) neither exact nor separable

20

(5) Given the initial value problem:

$$y''+xy'-2y=0, \qquad y(1)=1, \quad y'(1)=2$$

The problem has a unique solution of the form $\sum_{n=0}^{\infty}a_n(x-1)^n$ valid for $|x-1|<R$ Where a_0, a_1 and are given by:

(a) $a_0=a_1=1;\quad R=\infty$

$\boxed{\text{(b) } a_0=1, a_1=2;\quad R=\infty}$

(c) $a_0=a_1=1;\quad R=1$

(d) $a_0=1, a_1=2;\quad R=1$

$$\frac{a_{n+1}(x-1)^{\cancel{n+1}}}{a_n\cancel{(x-1)^n}}=|x-1|\left(\frac{a_{n+1}}{a_n}\right)$$

Part Two (answer the following questions and put it in the shown box)

(1) The solution of initial value problem $x' = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} x$, $x(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ is

$\begin{vmatrix} 2-r & 0 \\ -1 & 3-r \end{vmatrix} = 0 \qquad 6 - 2r - 3r + r^2 = 0 \qquad r^2 - 5r + 6 = 0$

$r = 2 \qquad (r-3)(r-2) = 0 \qquad r = 2, 3$

$\begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad -k_1 + k_2 = 0 \qquad k_2 = k_1 \qquad \boxed{k_1 = 1} \quad \boxed{k_2 = 1}$

$X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$

$\boxed{r = 3} \quad \begin{pmatrix} -1 & 0 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$??

$\begin{pmatrix} -1 & 0 \\ -1 & 0 \end{pmatrix} (k_1 \quad k_2)$

$-k_1 \quad -k_2 = 0$

$k_2 = -k_1$

$c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{3t}$

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Answer: $X = 2e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + -1 e^{3t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ or ~~No solution~~

(2) Given that the matrix $A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$, find the general solution of $x' = Ax$.

$\begin{pmatrix} 1-r & 1 \\ -1 & 1-r \end{pmatrix} = 1 - r - r + r^2 + 1 = 0 \qquad r^2 - 2r + 2 = 0$

$r = \frac{2 \pm \sqrt{4-8}}{2} = 1 \pm i$

$\begin{pmatrix} -i & 1 \\ -1 & -i \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad -ik_1 + k_2 = 0 \qquad k_2 = ik_1$

$\begin{matrix} k_1 = \\ k_2 = \end{matrix} \begin{pmatrix} 1 \\ i \end{pmatrix}$

$\begin{pmatrix} 1 \\ i \end{pmatrix} e^{t} (\cos t + i \sin t) =$

$e^{t} \begin{pmatrix} \cos t + i \sin t \\ -\sin t + i \cos t \end{pmatrix} = u + iv$

$y = c_1 u + c_2 v = \begin{pmatrix} c_1 e^{t} \cos t \\ -c_1 e^{t} \sin t \end{pmatrix} + \begin{pmatrix} c_2 e^{t} \sin t \\ c_2 e^{t} \cos t \end{pmatrix}$

Answer: $y = c_1 \begin{pmatrix} e^{t} \cos t \\ -e^{t} \sin t \end{pmatrix} + c_2 \begin{pmatrix} e^{t} \sin t \\ e^{t} \cos t \end{pmatrix}$

(3) If the general solution of the homogeneous system $x' = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} x$ is given by

$X_H(t) = C_1 e^{t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, then the general solution of the nonhomogeneous

system $X' = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} X + e^{-2t} \begin{pmatrix} -3 \\ -4 \end{pmatrix}$ is

$$\frac{dx}{dt} = x - 3e^{-2t}$$

$$\frac{dy}{dt} = 2y - 4e^{-2t}$$

$$x = C_1 e^{t} \Rightarrow \frac{dx}{dt} = C_1 e^{t} - 3e^{-2t}$$

$$x = C_1 e^{t} + \frac{3}{2} e^{-2t} + K_1$$

$$y = C_2 e^{2t} \quad \frac{dy}{dt} = 2C_2 e^{2t} - 4e^{-2t} \Rightarrow y = C_2 e^{2t} + 2e^{-2t} + K_2$$

$$X_p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{2t} + e^{-2t} \begin{pmatrix} \frac{3}{2} \\ 2 \end{pmatrix} + \begin{pmatrix} K_1 \\ K_2 \end{pmatrix}$$

Answer: $X_g = X_H + X_p = C_1 e^{t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{2t} + e^{-2t} \begin{pmatrix} \frac{3}{2} \\ 2 \end{pmatrix} \right) + \begin{pmatrix} K_1 \\ K_2 \end{pmatrix}$

(4) $f(t) = 1 + u_2(t)(t-2) + u_4(t)(2-t) + u_6(t)t^2$ The values of $f(3)$, $f(5)$ and $f(7)$ are

	2	4	6
1	1, $t-2$	1, $t-2$, $2-t$	1, $t-2$, $2-t$, t^2
1	$t-1$	1	$1+t^2$

$$f(t) = \begin{cases} 1 & t < u_2(t) \\ t-1 & u_2(t) \le t < u_4(t) \\ 1 & u_4(t) < t < u_6(t) \\ 1+t^2 & t > u_6(t) \end{cases}$$

Answer: $f(3) = 2$, $f(5) = 1$, $f(7) = 50$

(5) Compute

$$\int_0^t (t-\tau)\tau\, d\tau =$$

$$\int_0^t (t\tau - \tau^2)\, dt = t\frac{\tau^2}{2} - \frac{\tau^3}{3} \Big|_0^t$$

$$= \frac{t^3}{2} - \frac{t^3}{3} - 0 = \frac{t^3}{6}$$

Answer: $\frac{t^3}{6}$

(6) $f(t) = \int_0^t (t-\tau)^3 \cos(3\tau)\, d\tau$

Find the Laplace transform of $f(t)$.

$$\int_0^t (t-\tau)^3 \cos(3\tau)\, d\tau = t^3 * \cos 3t = f(t)$$

$$L(f(t)) = L(t^3) \cdot L(\cos 3t)$$

$$= \frac{3!}{s^4} \cdot \frac{s}{s^2+9} = \frac{6s}{s^4(s^2+9)}$$

Answer: $\frac{6s}{s^4(s^2+9)} = L(f(t)) = F(s)$

(7) What is the solution of initial value problem?

$$y'' = \delta(t-\pi) - \delta(t-2\pi); \qquad y(0) = y'(0) = 0$$

$$s^2 L(y) - 0 - 0 = e^{-\pi s} - e^{-2\pi s}$$

$$L(y) = \frac{e^{-\pi s} - e^{-2\pi s}}{s^2} = \left(e^{-\pi s} - e^{-2\pi s}\right) \cdot \frac{1}{s^2}$$

$$\mathcal{L}^{-1}\left(\frac{1}{s^2}\right) = t$$

$$y = u_\pi(t)(t-\pi) - u_{2\pi}(t)(t-2\pi)$$

Answer: $y = (t-\pi)u_\pi(t) - (t-2\pi)u_{2\pi}(t)$

(8) The inverse Laplace transform of

$$F(s) = \frac{se^{-2s}}{s^2+4s+13}$$

$$G(s) = \frac{s}{s^2+4s+13} = \frac{s}{(s+2)^2+9}$$

$$= \frac{s+2}{(s+2)^2+9} - \frac{2}{(s+2)^2+9}$$

$$g(t) = e^{-2t}\left(\cos 3t - \frac{2}{3}\sin 3t\right)$$

$$f(t) = u_2(t)\, e^{-2(t-2)}\left(\cos 3(t-2) - \frac{2}{3}\sin 3(t-2)\right)$$

Answer: $u_2(t)\, e^{-2(t-2)}\left(\cos 3(t-2) - \frac{2}{3}\sin 3(t-2)\right)$.

(9) Solve the initial value problem

$$y''+y=f(t); \quad y(0)=0, \quad y'(0)=1$$

Where

$$f(t)=\begin{cases}0 & t<2\\ 3 & 2\le t\end{cases} = 3\,u_2(t)$$

$s^2 L(y) - 1 + L(y) = \frac{3e^{-2s}}{s}$

$L(y)(s^2+1) = \frac{3e^{-2s}}{s} + 1 \Rightarrow L(y) = \frac{3e^{-2s}}{s(s^2+1)} + \frac{1}{s^2+1}$

$G(s) = \frac{1}{s(s^2+1)}$

$G(s) = \frac{1}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+D}{s^2+1} \Rightarrow As^2 + A + Bs^2 + Ds = 1$

$\boxed{A=1}$ $\boxed{D=0}$ $A+B=0$ $\boxed{B=-1}$ $G(s) = \frac{1}{s} - \frac{s}{s^2+1}$ $g(t) = 1-\cos t$

$y = 3u_2(t)(1-\cos(t-2)) + \sin t$

Answer: $3u_2(t)(1-\cos(t-2)) + \sin t$

(10) Determine $y''(1)$ and $y'''(1)$ if $y(x)$ satisfies

$$x^2y''+(1+x)y'+3(\ln(x))y=0; \quad y(1)=2, \quad y'(1)=1$$

$x^2y'' = -(1+x)y' - 3(\ln x)y$

$\boxed{y''(1) = -2} - 0$

$x^2y''' + 2xy'' = -(1+x)y'' - y' - \frac{3y}{x} - 3\ln x\, y'$

$y'''(1) - 4 = +4 - 1 - 6 - 0$

$y''' = 1$

Answer: $y''(1) = -2$ $y'''(1) = 1$

(11) In finding the power series solution $\sum_{n=0}^{\infty} a_n x^n$ of

$$y''+xy'+2y=0,$$

What is the recurrence relation?

$$\sum_{0}^{\infty} a_n x^n + \sum_{1}^{\infty} n a_n x^n + \sum_{2}^{\infty} n(n-1) a_n x^{n-2}$$

$$\downarrow \sum_{0}^{\infty} (n+2)(n+1) a_{n+2} x^n$$

$n=0$ $\quad 2a_0 + 2a_2 = 0 \quad \boxed{a_2 = \frac{-a_0}{2}}$

$$\sum_{1}^{\infty} \left[(n+2) a_n + (n+2)(n+1) a_{n+2} \right] x^n$$

$$\boxed{a_{n+2} = \frac{-a_n}{n+1} \qquad n \geq 1}$$

3

Answer: $a_{n+2} = \frac{-a_n}{n+1}$ $n \geq 1$ $\boxed{a_2 = \frac{-a_0}{2}}$

(12) What is the value of $y(3)$ if $y(x)$ is the solution of

$$x^2y''+4xy'+2y=0, \quad y(1)=1. \quad y'(1)=2$$

$r^2 + 3r + 2 = 0 \qquad (r+2)(r+1) = 0 \qquad r = -2, -1$

$y = c_1 e^{-2\ln x} + c_2 e^{-\ln x} = \frac{c_1}{x^2} + \frac{c_2}{x} = c_1 x^{-2} + c_2 x^{-1}$

$y(1) = \boxed{c_1 + c_2 = 1} \qquad y'(1) = \frac{-2c_1}{x^3} - \frac{c_2}{x^2} = -2c_1 - c_2 = 2$

$\boxed{c_2 = -2c_1 - 2}$

$c_1 - 2c_1 - 2 = 1 \Rightarrow -c_1 - 2 = 1 \Rightarrow \boxed{c_1 = -3} \qquad \boxed{c_2 = 4}$

$y = \frac{-3}{x^2} + \frac{4}{x}$

Answer: $y(3) = -\frac{1}{3} + \frac{4}{3} = 1$

(13) The differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=4x^{-1}$$

is a non-homogeneous Euler equation. Use variation of parameters to find a particular solution

$r^2 - 1 = 0$ $\boxed{r = 1, -1}$

$y_h = c_1 e^{\ln x} + c_2 e^{-\ln x} = c_1 x + \frac{c_2}{x}$

$y_p = u_1 x + u_2 \frac{1}{x}$

$\begin{pmatrix} x & \frac{1}{x} \\ 1 & -\frac{1}{x^2} \end{pmatrix} \begin{pmatrix} u_1' \\ u_2' \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{4}{x} \end{pmatrix}$

$w = -\frac{1}{x} - \frac{1}{x} = -\frac{2}{x}$

$u_1' = \frac{-\frac{4}{x}\cdot\frac{1}{x}}{-\frac{2}{x}} = \frac{2}{x}$ $\quad u_1 = 2\ln x$

$u_2' = \frac{x\cdot\frac{4}{x}}{-\frac{2}{x}} = -2x$ $\quad u_2 = -x^2$

$y_p = 2x\ln x - \frac{x^2}{x} = x\ln x^2 - x$

1 Answer: $y_p = x\ln x^2 - x$

$\boxed{(14)}$ Find the Laplace transform of

$t + g(t) = \begin{cases} 0 & 0<t<1 \\ 2-2t & 1<t<2 \\ -t & t>2 \end{cases}$

$$f(t)=\begin{cases} 0 & \text{if} \quad t\le 0 \\ t & \text{if} \quad 0<t\le 1 \\ 2-t & \text{if} \quad 1<t\le 2 \\ 0 & \text{if} \quad t>2 \end{cases}$$

$t(1 - u_1(t))$

$(2-t)(u_1 - u_2(t))$

t	1	2
t	2 − t	$\cancel{t}$
t	2	2

$f(t) = t - tu_1(t) + 2u_1(t) - 2u_2(t) - tu_1(t) + tu_2(t)$

$= t + u_1(t)(-t+2-t) + u_2(t)(-2+t) =$

$t + u_1(t)(2-2t) + u_2(t)(t-2)$

$(-2-2(t-1+1)) = (0 - 2(t-1))u_1(t)$

Answer: $F(s) = \frac{1}{s^2} + e^{-s}\left(-\frac{2}{s^2}\right) + e^{-2s}\left(\frac{1}{s^2}\right)$

$F(s) = \frac{1}{s^2} + e^{-s}\left(-\frac{2}{s^2}\right) + e^{-2s}\left(\frac{1}{s^2}\right)$

(15) The minimum radius of convergence of the power series solution of the ODE

$$(x^2-2x+2)y''+xy'+\frac{1}{x+2}y=0 \text{ about } x_0=2 \text{ is}$$

Problem points: $(x^2-2x+2)=0$ $\quad \frac{+2\pm\sqrt{4-8}}{2}=1\pm i$

$x+2=0$ $\quad \boxed{x=-2}$

$\rho_1=\sqrt{1^2+1^2}=\sqrt{2}$ $\quad (1+i)$

$\rho_2=\sqrt{1+1^2}=\sqrt{2}$ $\quad (1-i)$

$\rho_3=\sqrt{4^2+0}=4$ $\quad (-2)$ $\quad \rho_{min}(\sqrt{2},\sqrt{2},4)$

Answer: $\rho_{minimum}=\sqrt{2}$

(16) Solve the initial value problem

$$\frac{d^2y}{dt^2}\frac{dy}{dt}=3,\quad y(1)=1,\quad y'(1)=1. \text{ Then } y(5) \text{ is}$$

missing y

$y''y'=3$ $\quad V=y'$ $\quad V'=y''$

$VV'=3 \Rightarrow V\frac{dV}{dt}=3 \Rightarrow V\,dV=3\,dt$

$\frac{V^2}{2}=3t+C \Rightarrow V^2=6t+C_1 \Rightarrow y'=\sqrt{6t+c}$

$\frac{dy}{dt}=\sqrt{6t+c} \Rightarrow dy=\sqrt{6t+c}\,dt$ $\quad y=\int(6t+c)^{\frac{1}{2}}dt$

$y=\frac{(6t+c)^{\frac{3}{2}}}{6\times\frac{3}{2}}=\frac{(6t+c)^{\frac{3}{2}}}{9}+C_2=\frac{\sqrt{(6t-5)^3}}{9}+C_2$

Answer: $y(5)=\frac{\sqrt{(25)^3}}{9}+\frac{8}{9}=\frac{125}{9}+\frac{8}{9}=\frac{133}{9}=14.78$

$y'(1)=\sqrt{6+c}=1$ $\quad 6+c=1$ $\quad y=\frac{\sqrt{(6t-5)^3}}{9}+\frac{8}{9}$

$\boxed{c=-5}$

$y(1)=1=\frac{\sqrt{(1)^3}}{9}+C_2 \Rightarrow C_2=1-\frac{1}{9}=\frac{8}{9}$

(17) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$$

$y'' - 4y' + 13y = 0$ $\quad r^2 - 4r + 13 = 0$

$r = \dfrac{+4 \pm \sqrt{16 - 52}}{2} = 2 \pm 3i$

$y = e^{2t}(c_1 \cos 3t + c_2 \sin 3t)$

Answer: $e^{2t}(c_1 \cos 3t + c_2 \sin 3t)$

(18) Using the method of undetermined coefficient, find a suitable form for a particular solution of the differential equation:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5 = e^{-2x}\cos x$$

$y'' + 4y' + 5 = e^{-2x}\cos x \Rightarrow y'' + 4y' = e^{-2x}\cos x - 5$

$r^2 + 4r = 0 \quad r(r+4) = 0 \quad r = 0, -4$

$y_h = c_1 + c_2 e^{-4t}$

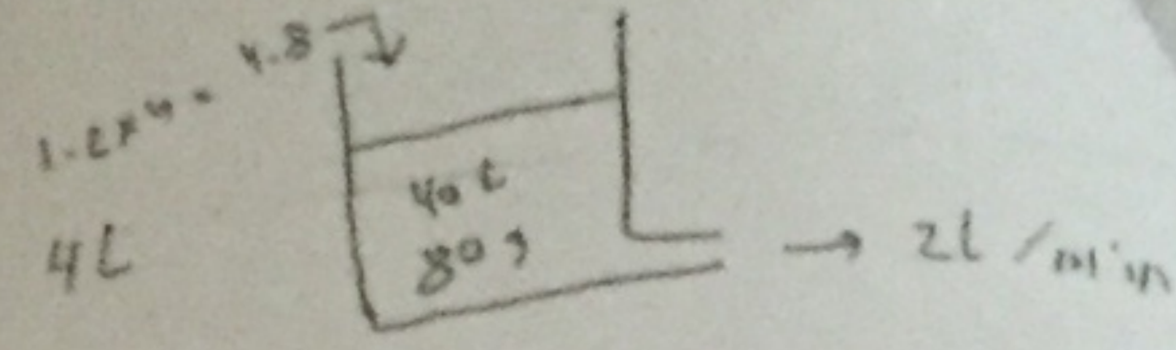
$y_{p_1} = e^{-2x}(A\cos x + B\sin x)$

$y_{p_2} = t^1 D = Dt$

$y_p = D\cancel{x} + e^{-2x}(A\cos x + B\sin x)$

Answer: $y_p = Dx + e^{-2x}(A\cos x + B\sin x)$

$y = y_h + y_p$



(19) A very large tank contains 40 L brine of concentration 2g/L salt. Brine of concentration 1.2 g/L salt flows into the tank at 4L/min and the well-mixed solution is pumped out at 2 L/min. Assuming that the tank does not overflow what is the concentration of salt in the tank after 10 min? (Choose the closest value.)

$\frac{dq}{dt}$ = rate in − rate out = $4.8 - \frac{2q}{40+2t}$ = $4.8 - \frac{q}{20+t}$

I.V.P.

$\frac{dq}{dt} + \frac{q}{20+t} = 4.8$

$q(0) = 80$ g

$\frac{dq}{dt} + \frac{q}{20+t} = 4.8 \qquad \mu(t) = e^{\int \frac{1}{20+t}dt} = 20+t$

$q(t) = \frac{\int 4.8(20+t)\,dt + c}{20+t} = \frac{96t + 2.4t^2 + c}{20+t} \qquad q(t) = \frac{2.4t^2 + 96t + 1600}{20+t}$

$q(0) = 80 = \frac{c}{20} \Rightarrow c = 1600$

Answer: $q(10) = \frac{240 + 960 + 1600}{30} = 93.3$ g

(20) Find a differential equation of 1[st]. order whose integrating factor is e^y

$\frac{dx}{dy} + x = 4y$

$\mu(y) = e^{\int 1\,dy} = e^y$

$x = e^{-y}\int 4y e^y\,dy + c = e^{-y}(4ye^y - 4e^y + c)$

$x = 4y - 4 + c$

Answer: $\frac{dx}{dy} + x = 4y$ L.D.E of x with $\mu(y) = e^y$

(5) Compute

$$\int_0^t (t-\tau)\tau\, d\tau =$$

(a) 0
(b) $\frac{1}{2}t^2$
(c) $\frac{1}{6}t^3$
(d) $-\frac{1}{2}\tau^3$
(e) $\frac{1}{2}(t-\tau)^3$

$= \int_0^t t\tau - \tau^2\, d\tau$

$= t\frac{\tau^2}{2}\Big|_0^t - \frac{\tau^3}{3}\Big|_0^t$

$= \frac{t^3}{2} - \frac{t^3}{3} = 0$

$t-\tau \quad \tau$

$\tau \quad (t-\tau)$

$1 \quad \frac{(t-\tau)^{1+1}}{-2} \quad \frac{(t-\tau)^2}{-2}$

$0 \quad \frac{(t-\tau)^3}{6}$

$\tau\frac{(t-\tau)^2}{-2} - \frac{(t-\tau)^3}{6}$

$\frac{t\tau - \tau^2}{-2}\Big|^t$

(6) Indicate the type of the following differential equation:

$$\sqrt{x+y} + \frac{dy}{dx} = 0$$

(a) Exact
(b) Linear
(c) Separable
(d) Homogeneous
(e) None of the above

$dy = \sqrt{x+y}\, dx$

$\frac{dy}{dx} = -\sqrt{x+y}$

$dy = -\sqrt{x+y}\, dx$

non-exact

$M_y = \frac{1}{2\sqrt{x+y}}$ $\quad \frac{dy}{dx}$

$N_x = 0$ $\quad \frac{1}{2\sqrt{x+y}}$

$(y')^2 = -x-y$

$\cancel{y}^2 = (y')^2 = -x-y$

$\cancel{\frac{(dy)^2}{(dx)^2} = -x-y}$

$\cancel{\frac{(dy)^2}{y} = -x(dx)^2}$

(7) What is the solution of initial value problem?

$y'' = \delta(t-\pi) - \delta(t-2\pi)$; $y(0) = y'(0) = 0$

(a) $y(t) = u_\pi(t)t - u_{2\pi} + \pi$

(b) $y(t) = \frac{1}{t^2}(e^{-\pi}t - e^{-2\pi t})$

(c) $y(t) = c_1 + c_2 t$

(d) $y(t) = u_\pi(t)(t-\pi) - u_{2\pi}(t)(t-2\pi)$

(e) $y(t) = u_\pi(t)t - u_{2\pi}t$

$L(y'') = e^{-\pi s} - e^{-2\pi s}$

$s^2 L(y) - sy(0) - y'(0) = e^{-\pi s} - e^{-2\pi s}$

$s^2 L(y) = e^{-\pi s} - e^{-2\pi s}$

$L(y) = \frac{e^{-\pi s}}{s^2} - \frac{e^{-2\pi s}}{s^2}$

$y = (t-\pi) u_\pi(t) - (t-2\pi) u_{2\pi}!$

(8) The inverse Laplace transform of $F(s) = \frac{se^{-2s}}{s^2+4s+13}$

(a) $u_2(t)(e^{-2t}\cos(3(t-2)) - e^{-2t}\sin(3(t-2)))$

(b) $u_2(t)((e^{-2(t-2)}\cos(3(t-2)) - \frac{2}{3}e^{-2(t-2)}\sin(3(t-2)))$

(c) $u_2(t)(e^{-2t}\cos(3t) - \frac{2}{3}e^{-2t}\sin(3t))$

(d) $u_2(t)(e^{-2(t+2)}\cos(3(t+2)) - \frac{2}{3}e^{-2(t+2)}\sin(3(t+2))$

(e) $e^{-2(t+2)}\cos(3(t+2)) - \frac{2}{3}e^{-2(t+2)}\sin(3(t+2))$

$\frac{se^{-2s}}{s^2+4s+4-4+13}$

$= \frac{se^{-2s}}{\cancel{s^2+4s} \ (s+2)^2+9}$

$= \frac{(s+2-2)e^{-2s}}{(s+2)^2+9}$

$\frac{(s+2)}{(s+2)^2+9}e^{-2s} - \frac{2\times 3 e^{-2s}}{3\,(s+2)^2+9}$

$= e^{-2(t-2)}\cos 3(t-2)\, u_2(t) - \frac{2}{3} e^{-2(t-2)} \sin \ldots u_2(t)$

(9) Solve the initial value problem

Where $y''+y=f(t); \quad y(0)=0, \ y'(0)=1$

$f(t) \doteq 3\, u_2(t)$

$$f(t)=\begin{cases}0 & t<2\\ 3 & 2\le t\end{cases}$$

$L(y'')+L(y) = L(f(t))$

$= s^2 L(y) - sy(0) - y'(0) + L(y) = 3$

$s^2 L(y) - 1 + L(y) = 3\frac{e^{-2s}}{s}$

$s^2 L(y) + L(y) = \frac{3e^{-2s}+2}{s}$

$L(y) = \frac{3e^{-2s}+2}{2(s^2+1)}$

$y = \frac{1}{s^2+1} + \frac{3}{2}\frac{e^{-2s}}{(s^2+1)}$

$= \sin t + \frac{3}{2}u_2(t)\sin$

(a) $y(t)=\sin(t)+3(t-2)u_2(t)-3u_2(t)\cos(t-2)$

(b) $y(t)=c_1\cos(t)+c_2\sin(t)$

(c) $y(t)=\sin(t)+3u_2(t)+3u_2(t)\sin(t)$

(d) $y(t)=\sin(t)+3u_2(t)+3u_2(t)\sin(t-2)$

(e) $y(t)=\sin(t)+3u_2(t)-3u_2(t)\cos(t-2)$

$s^2 L(y) + L(y) = \frac{3e^{-2s}}{s} + 1$

$L(y) = \frac{3e^{-2s}}{s(s^2+1)} + \frac{1}{s^2+1} =$

$y =$

$\frac{A}{s} + \frac{Bs+C}{s^2+1} = \frac{3}{(s^2+1)(s)}$

$A(s^2+1) + s(Bs+C) = 3$

$A = 3$

$s=1 \Rightarrow 2A + B + C = 3$

$B + C = -3$

$s=-1 \quad 2A - (-B+C) = 3$

$6 + B - C = 3$

$B - C = -3$

$B + C = 3$

$B = 0, C = 3$

$\frac{3}{s} + \frac{3}{s^2+1}$

$3s^2+3+3s$

(10) Determine $y''(1)$ and $y'''(1)$ if $y(x)$ satisfies

$$x^2y''+(1+x)y'+3(\ln(x))y=0; \quad y(1)=2, \quad y'(1)=0$$

(a) $y''(1)=0; \quad y'''(1)=6$

(b) $y''(1)=1; \quad y'''(1)=-6$

(c) $y''(1)=1; \quad y'''(1)=6$

(d) $y''(1)=0; \quad y'''(1)=1$

(e) $y''(1)=0; \quad y'''(1)=-6$

$x^2 y'' = -(1+x)y' - 3(\ln(x))y$

$y''(1)$

$\Rightarrow y'' = -2y'(1) - 3(0)$

$= 0$

y''': $2xy'' + x^2y''' = -(y' + (1+x)y'') - 3(\frac{1}{x}y + \ln x\, y')$

$0 + y''' = (0+0) - 3(2)$

$$\mathcal{L}(y) = 3\left(\frac{1}{s} - \frac{s}{s^2+1}\right)e^{-2s} + \frac{1}{s^2+1}$$

$$\left(\frac{3}{s} - \frac{3s}{s^2+1}\right)e^{-2s} + \frac{1}{s^2+1}$$

9. $y = 3\,u_2(t) - u_2(t)\,3\cos(t-2) \quad \sin t$

$$\frac{3}{s} + \frac{3}{s^2+1}\Big)e^{-2s} + \frac{1}{s^2+1}$$

$$\frac{3s^2+3+3s}{s(s^2+1)}$$

$y'(0)=1$

$y(0)=0$

$y =$

$$\mathcal{L}(y'') + \mathcal{L}(y) = 3\frac{e^{-2s}}{s}$$

$$s^2\mathcal{L}(y) - sy(0) - y'(0) + \mathcal{L}(y) = 3\frac{e^{-2s}}{s}$$

$$s^2\mathcal{L}(y) - 1 + \mathcal{L}(y) = 3\frac{e^{-2s}}{s}$$

$2 \quad -1+C=1$

$$\mathcal{L}(y)(s^2+1) = 3\frac{e^{-2s}}{s} + 1$$

2

$1+C=1$

$C=0$

$$= \frac{3}{s(s^2+1)}e^{-2s} + \frac{1}{s^2+1}$$

$$\frac{A}{s} + \frac{Bs+C}{s^2+1} = 1$$

$A(s^2+1) + s(Bs+C) = 1$

$A = 1$

$2A + B + C = 1$

$2A + B + C = 1$

$\Rightarrow \quad 4A + 2B = 2$

$4 + 2B = 2$

$2B = -2$

$B = -1$

$-1-1=-2$

(11) The differential equation

$c_1 x^r + c_2 x$

$c_1 x + c_2 x^{-1}$

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=4x^{-1}$$

$r^2+(\alpha-1)r-1=0$

$r=\pm 1 \quad y_h = c_1 e^t + c_2 e^{-t} = c_1 x + c_2 x^{-1}$

is a non-homogeneous Euler equation equation. Use variation of parameters to find a particular solution

(a) $x+x^{-1}$

(b) $x^{-1}\ln x$

(c) $\dfrac{-1-2\ln x}{x}$

(d) $1+2\ln x$

(e) None of the above.

$y_p = u_1(t)e^t + u_2(t)e^{-t}$

$W = \begin{vmatrix} e^t & e^{-t} \\ e^t & -e^{-t} \end{vmatrix} = -1-1=-2$

$W = ce^{-\int \frac{1}{x}dx} = \frac{c}{x}$

$u_1 = -\int \frac{y_2\, g(t)}{W_1}$

$u_1 = -\ln x$

$u_2 = \frac{1}{2}x^2$

$= -4\int \frac{e^{-t}}{t}$

$= -4e^{-t} \quad 4e^t$

$u_2 = \int \dfrac{e^t \frac{4}{t}}{\frac{1}{t}} = 4e^t$

$-4\int x^{-1}$

$-4\ln x$

$y_2 = 4\frac{x^{r+1}}{r+1} \quad \frac{4x^2}{2}$

$x\ln x + 2x^2x^{-1}$

$x\ln x + 2x$ (12) Let $y=(x)=\sum_{n=0}^{\infty}a_nx^n$. The recurrence relation reads

$$a_2=-a_0/2, \quad a_{n+2}=\frac{n}{n+2}a_{n+1}-\frac{a_n}{(n+2)(n+1)}, \quad n=1,2,3,\ldots$$

If $a_0=1, a_1=2,$ the first four terms of the series solution is

(a) $y(x)=1+2x+3x^2+4x^3+\ldots$

(b) $y(x)=1+2x-\frac{1}{2}x^2-4x^2+\ldots$

(c) $y(x)=1+2x-\frac{1}{2}x^2-\frac{1}{2}x^3+\ldots$

(d) $y(x)=1+2x-\frac{1}{2}x^2-\frac{1}{4}x^3+\ldots$

(e) $y(x)=1+2x-3x^2-4x^3+\ldots$

$y = a_0 + a_1x + a_2x^2 + a_3x^3$

$a_2 = -\frac{1}{2}$

$a_3 = \frac{a_2}{3} - \frac{a_1}{6} = \frac{-1}{6} - \frac{2}{6} = -\frac{3}{6} = -\frac{1}{2}$

$1+2x \quad -\frac{1}{x}$

$4\ln x$

$r^2-1=0$

$y = c_1 x + c_2 x^{-1}$

$W = ce^{-\int \frac{1}{x}} = ce^{-\ln x} = \frac{1}{x}$

$4x^{-3} \quad 4x^{-3} \quad 4\frac{x^{-3+1}}{r+1}$

$-\frac{2}{x^2}x + \frac{4\ln x}{x}$

$- \quad c_1 x + c_2$

(13) What is the value of $y(3)$ if $y(x)$ is the solution of

$$x^2y''+4xy'+2y=0, \quad y(1)=1. \quad y'(1)=2$$

$2y = -x^2y'' - 4xy'$

$2(1) = -y''(1) - 4(2)$

$2 = -y''(1) - 8$

$y''(1) = -10$

(a) -2
(b) -1
(c) 0
(d) 1
(e) 2

$r^2+3r+2=0$

$(r+2)(r+1)=0$

$y = c_1e^{-2t} + c_2e^{-t}$

$= c_1x^{-2} + c_2x^{-1}$

$y' = -2c_1e^{-2t} - c_2e^{-t}$

$2 = -2c_1e^{-2} - c_2e^{-1}$

$1 = c_1e^{-2} + c_2e^{-1}$

$2 = -2c_1e^{-2} - c_2e^{-1}$

$-c_1e^{-2} = 3$

$c_1 = \frac{-3}{e^{-2}} = -3e^2$

$1 = -3 + c_2e^{-1}$

$4 = c_2e^{-1}$

$c_2 = 4e$

$y = -3e^2e^{-2t} + 4ee^{-t}$

$y_3 = -3e^2e^{-6} + 4ee^{-3}$

$= -3e^{-3} + 4e^{-2}$

$y = c_1x^{-2} + c_2x^{-1}$

$1 = c_1 + c_2$

$y' = -2c_1x^{-3} - c_2$

$-2c_1 - c_2 =$

$-c_1 = 3 \quad c_1 =$

$y = c_1x^{-2} + c_2x^{-1}$

$1 = c_1 + c_2$

$y' = -2c_1x^{-3} - c_2$

$2 = -2c_1 - c_2$

$3 = -c_1$

$c_1 = 3$

$1 = 3 + c_2 \Rightarrow c_2 =$

$y = 3x^{-2} + -2x^{-1}$

$= \frac{3}{x^2} - \frac{2}{x}$

$y(3) = \frac{3}{9} - \frac{2}{3}$

$= \frac{3}{9} - \frac{6}{9} = -\frac{3}{9} = -\frac{1}{3}$

$y(3) = -1$

(14) Classify the singular points of the given equation

$$x^2(2-x^2)y''+(2/x)y'+4y=0$$

$x^2(2-x^2)=0$

$x = 0, 0, \sqrt{2}, -\sqrt{2}$

5

The regular singular points are $\sqrt{2}, -\sqrt{2}$

The irregular singular points are $0, 0$

$\lim_{x\to 0} x \frac{2}{x^3(2-x^2)} = \frac{2}{0} = \infty$

$\lim_{x\to 0} x^2 \frac{4}{x^2(2-x^2)} = 2$

$\lim_{x\to\sqrt{2}} -\frac{(\sqrt{2}-x)}{(\sqrt{2}-x)} \frac{2}{x^3(\sqrt{2}}$

$=$

Ex. Find the soln.

$$\underset{\sim}{X}' = \begin{bmatrix} 1 & 5 \\ -1 & -1 \end{bmatrix} \underset{\sim}{X} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

soln. $\underset{\sim}{X} = c_1 \begin{bmatrix} -\cos 2t + 2\sin 2t \\ 2\cos 2t \end{bmatrix} + c_2 \begin{bmatrix} -2\cos 2t - \sin 2t \\ \sin 2t \end{bmatrix} + \begin{bmatrix} 5/2 \\ -1/2 \end{bmatrix}$

Ex. 2

$$\underset{\sim}{X}' = A\underset{\sim}{X} + B$$

$$A = \begin{bmatrix} 2 & 2 \\ -3 & -3 \end{bmatrix} \quad B(t) = \begin{bmatrix} 1 \\ 2t \end{bmatrix}$$

Soln.

$$\underset{\sim}{X} = \begin{bmatrix} c_1 + 2c_2 e^{-t} + 2 - t + 2t^2 \\ -c_1 - 3c_2 e^{-t} - 3 + 3t - 2t^2 \end{bmatrix}$$

(15) Find m such that y^m is an integrating factor of the ODE:

$$-2xy+(2x^2+y)\frac{dy}{dx}=0$$

(a) -2
(b) 2
(c) 3
(d) -3
(e) None of the above.

$-2xy\,dx + (2x^2+y)\,dy = 0$

$M_y = -2x$

$N_x = 4x$

$Q = \frac{N_x - M_y}{M} = \frac{4x+2x}{-2xy} = \frac{6x}{2xy} = \frac{3}{y}$

$m = e^{\int \frac{3}{y} dy} = e^{3\ln y} = y^3$

(16) $$f(t)=\int_0^t (t-\tau)^3\cos(3\tau)\,d\tau$$

Find the Laplace transform of $f(t)$.

(a) $\frac{6s}{s^4(s^2+9)}$ ✓

(b) $\frac{18s}{s^4(s^2+9)}$

(c) $\frac{6}{s^4}+\frac{3s}{s^2+9}$

(d) $\frac{6}{s^4(s^2+9)}$

(e) $\frac{s}{s^4(s^2+9)}$

$L(t^3 * \cos 3t)$

$= L(t^3)\cdot L(\cos 3t)$

$= \frac{3!}{s^4}\cdot\frac{s}{s^2+9}$

U.V.C.

Ex $\dot{x}_1 = x_1 + 3x_2 + \sin t$
$\dot{x}_2 = x_1 - x_2 - \cos t$ $\Big\}\rightarrow$ (*)

$X = c_1 X^{(1)} + c_2 X^{(2)}$
$= \begin{bmatrix} 3c_1 e^{2t} - c_2 e^{-2t} \\ c_1 e^{2t} + c_2 e^{-2t} \end{bmatrix}$
$= c_1 \begin{bmatrix} 3 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-2t}$

$g(t) = \begin{bmatrix} \sin t \\ -\cos t \end{bmatrix}$

$X_p = \begin{bmatrix} A\sin t + B\cos t \\ C\sin t + D\cos t \end{bmatrix}$

put $x_1 = A\sin t + B\cos t$
$x_2 = C\sin t + D\cos t$
$\dot{x}_1 = A\cos t - B\sin t$
$\dot{x}_2 = C\cos t - D\sin t$

put in (*)

$A\cos t - B\sin t = A\sin t + B\cos t + 3C\sin t + 3D\cos t + \sin t$
$C\cos t - D\sin t = A\sin t + B\cos t - C\sin t - D\cos t - \cos t$

coefficient of cost & sint

Q1 cost: $A = B + 3D$
Q1 sint: $-B = A + 3C + 1$

Q2 cost: $C = B - D - 1$
Q2 sint: $-D = A - C$

$A + B + 3C = -1 \rightarrow$ (1)
$-A + B + 3D = 0 \rightarrow$ (2)
$A - C + D = 0 \rightarrow$ (3)
$B - C - D = 1 \rightarrow$ (4)

From (1)&(2) $2B + 3C + 3D = -1$
(3)+(4) $A + B - 2C = 1$
(1)-(4) $A + 4C + D = -2$
(2)+(3) $B + C + 4D = 0$

$B - 6C - D = 3$
$B - C + 4D = 0$
$-5C - 5D = 3$

$\left[\begin{array}{cccc|c} 1 & 1 & 3 & 0 & -1 \\ -1 & 1 & 0 & 3 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 \end{array}\right]$

$\begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} \frac{1}{5} \\ \frac{2}{5} \\ -\frac{2}{5} \\ -\frac{1}{5} \end{bmatrix}$

$X_p = \begin{bmatrix} -\frac{1}{5}\sin t + \frac{2}{5}\cos t \\ -\frac{2}{5}\sin t - \frac{1}{5}\cos t \end{bmatrix}$

$X_p = \ldots$

(17) Consider the point $x\frac{d^2y}{dx^2}+e^x\frac{dy}{dx}+y\cos x=0$

(a) $x_0=0$ is an ordinary point.

(b) $x_0=0$ is a regular singular point.

(c) $x_0=0$ is an irregular singular point.

(d) None of the above

$y'' + \frac{e^x}{x}y' + \frac{\cos x}{x}y = 0$

$\lim_{x\to 0} x\frac{e^x}{x} = e^0 = 1$

$\lim_{x\to 0} x^2\frac{e^x}{x} = 0$

(18) In finding the power series solution $\sum_{n=0}^{\infty} a_n x^n$ of

$y''+xy'+2y=0,$

What is the recurrence relation?

(a) $a_{n+2}=-a_n/(n+1), \quad n=0,1,2,\ldots$

(b) $a_{n+2}=a_n/(n+2), \quad n=0,1,2,\ldots$

(c) $(n+2)a_{n+2}-a_{n+1}-a_n=0, \quad n=0,1,2,\ldots$

(d) $a_{n+2}=a_n/((n+2)(n+1)), \quad n=0,1,2,\ldots$

(e) $a_2=-a_0/2; \quad (n+2)(n+1)a_{n+2}-n(n+1)a_{n+1}+a_n=0, \quad n=,1,2,\ldots$

$y' = \sum_{n=1} n a_n x^{n-1}$

$y'' = \sum_{n=2} n(n-1)a_n x$

$\sum_{n=0} (n+2)(n+1)a_{n+2}x^n + \sum_{n=1} n a_n x^n + 2\sum_{n=0} a_n$

$n=0$

$2a_2 + 2a_0 = 0$

$a_2 = -a_0$

$n=1$

$a_n(n+2)$

$(n+2)(n+1)a_{n+2} + n a_n + 2a_n = 0$

$(n+1)\cancel{(n+2)}a_{n+2} = -a_n\cancel{(n+2)}$

$a_{n+2} = \frac{-a_n}{n+1}$

Ex $\dot{\vec{x}} = A\vec{x} + B(t)$ $A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix}$ $B(t) = \begin{bmatrix} \sin t \\ -\cos t \end{bmatrix}$

$$\vec{x}_h = c_1 \begin{bmatrix} 3 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-2t}$$

$$= \begin{bmatrix} 3e^{2t} & -e^{-2t} \\ e^{2t} & e^{-2t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$= \Phi(t)\vec{c}$$

$3+1=4$

$\boxed{2}$ $|\Phi(t)| = \cancel{8}$

$$\Phi^{-1}(t) = \frac{1}{4} \begin{bmatrix} e^{-2t} & e^{-2t} \\ -e^{2t} & 3e^{2t} \end{bmatrix}$$

$$\Phi^{-1}(t)B(t) = \frac{1}{4} \begin{bmatrix} e^{-2t}\sin t - e^{-2t}\cos t \\ -e^{2t}\sin t \quad 3e^{2t}\cos t \end{bmatrix}$$

$$\int \Phi^{-1}(t)B(t)\,dt = \frac{1}{4}$$

(19)Given that the matrix $A=\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}$ has complex eigenvalues $r_1 = 1+i$ and $r_2 = 1-i$, with corresponding complex eigenvectors $\begin{pmatrix}1\\ i\end{pmatrix}$ and $\begin{pmatrix}1\\ -i\end{pmatrix}$, find the general solution of $x' = Ax$.

$c_1\begin{pmatrix}1\\ i\end{pmatrix}e^{t} + c_2\begin{pmatrix}1\\ +i\end{pmatrix}t\, e^{t} + c_2\begin{pmatrix}1\\ -i\end{pmatrix}e^{t}$

(a) $x = C_1e^{t}\begin{pmatrix}\cos(t)\\ -\sin(t)\end{pmatrix} + C_2e^{t}\begin{pmatrix}\sin(t)\\ \cos(t)\end{pmatrix}$

(b) $x = C_1e^{t}\begin{pmatrix}\cos(t)\\ -\sin(t)\end{pmatrix} + C_2e^{t}\begin{pmatrix}\sin(t)\\ -\cos(t)\end{pmatrix}$

(c) $x = C_1e^{t}\begin{pmatrix}\sin(t)\\ \cos(t)\end{pmatrix} + C_2e^{t}\begin{pmatrix}\sin(t)\\ -\cos(t)\end{pmatrix}$

(d) $x = C_1e^{t}\begin{pmatrix}\cos(t)\\ -\sin(t)\end{pmatrix} + C_2e^{t}\begin{pmatrix}\cos(t)\\ \sin(t)\end{pmatrix}$

(e) $x = C_1e^{t}\begin{pmatrix}2\cos(t)\\ \sin(t)\end{pmatrix} + C_2e^{t}\begin{pmatrix}\sin(t)\\ 2\cos(t)\end{pmatrix}$

$Y = C_1 K e^{rt}$

$= c_1\begin{pmatrix}1\\ i\end{pmatrix} e^{t+it} = c_1\begin{pmatrix}1\\ i\end{pmatrix}e^{t}(\cos t + i\sin t)$

$c_1e^{t}\begin{pmatrix}\cos t\\ -\sin t\end{pmatrix} + i\begin{pmatrix}\sin t\\ \cos t\end{pmatrix} = c_1 e^{t}\begin{pmatrix}\cos t + i\sin t\\ \cos t\, i - \sin t\end{pmatrix}$

(20) The solution of initial value problem $x' = \begin{pmatrix}2 & 0\\ -1 & 3\end{pmatrix}x$; $x(0) = \begin{pmatrix}1\\ 3\end{pmatrix}$ is

(a) $x(t) = e^{3t}\begin{pmatrix}0\\ 1\end{pmatrix} + e^{2t}\begin{pmatrix}1\\ 1\end{pmatrix}$

(b) $x(t) = 2e^{3t}\begin{pmatrix}0\\ 1\end{pmatrix} + e^{2t}\begin{pmatrix}1\\ 1\end{pmatrix}$

(c) $x(t) = 3e^{3t}\begin{pmatrix}0\\ 1\end{pmatrix} + e^{2t}\begin{pmatrix}1\\ 0\end{pmatrix}$

(d) $x(t) = 2e^{3t}\begin{pmatrix}1\\ 2\end{pmatrix} + e^{2t}\begin{pmatrix}1\\ 0\end{pmatrix}$

(e) $x(t) = e^{2t}\begin{pmatrix}1\\ 3\end{pmatrix}$

$\begin{pmatrix}2-r & 0\\ -1 & 3-r\end{pmatrix}$

$(2-r)(3-r)+1=0$

$6-3r-2r+r^2 \cancel{+1} = 0$

$r^2-5r+6=0$

$(r-2)(r+3)=0$

$r = 2, 3$

$X = c_1e^{2t}\begin{pmatrix}1\\ 1\end{pmatrix} + c_2e^{3t}\begin{pmatrix}0\\ 1\end{pmatrix}$

$\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}$

$\begin{pmatrix}c_1e^{2t} + 0\\ c_1e^{2t} + c_2e^{3t}\end{pmatrix} = \begin{pmatrix}1\\ 3\end{pmatrix}$

$-k_1 + k_2 = 0$

$k_1 = k_2$

$c_1e^{2t}\begin{pmatrix}1\\ 1\end{pmatrix}$

$c_1 = 1$

$1 + c_2 = 3$

$1e^{2t} + 2e^{3t}\begin{pmatrix}0\\ 1\end{pmatrix}$

-1

$\begin{pmatrix}-1 & 0\\ -1 & 0\end{pmatrix}$

$k_1 = 0$

$k_2 = 1$

$\begin{pmatrix}0\\ 1\end{pmatrix}$

Solve the nonhom. system by variation of parameters.

$$\dot{x} = Ax + B(t) \quad A = \begin{bmatrix} 1 & 0 \\ -2 & 2 \end{bmatrix} \quad B(t) = \begin{bmatrix} e^{-2t} \\ 0 \end{bmatrix}$$

$$x_h = c_1 x^{(1)} + c_2 x^{(2)}$$

$$= c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{2t}$$

$$= \begin{bmatrix} e^{t} & 0 \\ 2e^{t} & e^{2t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$= \Phi(t)\, \vec{c}$$

$\boxed{2}$ $|\Phi(t)| = e^{3t}$

$$\Phi^{-1}(t) = \frac{1}{e^{3t}} \begin{bmatrix} e^{2t} & 0 \\ -2e^{t} & e^{t} \end{bmatrix}$$

$$= \begin{bmatrix} e^{-t} & 0 \\ -2e^{-2t} & e^{-2t} \end{bmatrix}$$

$$\Phi^{-1}(t) B(t) = \begin{bmatrix} e^{-3t} \\ -2e^{-4t} \end{bmatrix}$$

$$\int \Phi^{-1}(t) B(t)\, dt = \begin{bmatrix} -\frac{1}{3} e^{-3t} \\ \frac{1}{2} e^{-4t} \end{bmatrix}$$

$$x_p = \Phi(t) \int \Phi^{-1}(t) B(t)\, dt$$

$$= \begin{bmatrix} e^{t} & 0 \\ 2e^{t} & e^{2t} \end{bmatrix} \begin{bmatrix} -\frac{1}{3} e^{-3t} \\ \frac{1}{2} e^{-4t} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{3} e^{-2t} \\ -\frac{2}{3} e^{-2t} + \frac{1}{2} e^{-2t} \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} \\ -\frac{1}{6} \end{bmatrix} e^{-2t}$$

$$= -\frac{1}{6} \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t}$$

$$x_g = x_h + x_p$$

$$= c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{2t} - \frac{1}{6} \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t}$$

$$-\frac{4}{6} + \frac{3}{6} = -\frac{7}{6}$$

Key

BIRZEIT UNIVERSITY

MATHEMATICS DEPARTMENT

SPRING 2006 MATH 331 FINAL EXAM

INSTRUCTORS: Choose either 1 or 2

1. RIMON JADOUN (SECTIONS 1 AND 2)
2. RASSEM KA'BI (SECTIONS 3 AND 4)

NAME in Arabic:

NUMBER:

SECTION:

TIME: 150 MINUTES

INSTRUCTIONS:

1. All answers to question one should be on page 2 of the test. After solving the problem write your suitable choice.
2. No calculators are allowed in this examination.
3. You should close your cellular phones before you enter this test, any use of phones or any other electronic tools will be considered cheating.
4. Questions are not allowed in this test, for any inconveniency, the instructor will respond to questions about typing 15 minutes after the test starts and only for 15 minutes.
5. The test consists of *three* questions in *SIX* pages.
6. Be careful, abide by the rules, don't cheat and don't talk to your neighbor.
7. When you write your name on the sheet given to you write the number you register your name on here.

OUR NUMBER:

GOOD LUCK

(3) If the general solution of the homogeneous system $x' = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} x$ is given by

$X_H(t) = C_1 e^{t}\begin{pmatrix}1\\0\end{pmatrix} + C_2 e^{2t}\begin{pmatrix}0\\1\end{pmatrix}$, then the general solution of the nonhomogeneous system $X' = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} X + e^{-2t}\begin{pmatrix}-3\\-4\end{pmatrix}$ is

(a) $x(t) = C_1 e^{t}\begin{pmatrix}1\\0\end{pmatrix} + C_2 e^{2t}\begin{pmatrix}0\\1\end{pmatrix} + e^{-t}\begin{pmatrix}4\\3\end{pmatrix}$

(b) $x(t) = C_1 e^{t}\begin{pmatrix}1\\0\end{pmatrix} + C_2 e^{2t}\begin{pmatrix}0\\1\end{pmatrix} + e^{-t}\begin{pmatrix}4\\0\end{pmatrix}$

(c) $x(t) = C_1 e^{t}\begin{pmatrix}1\\0\end{pmatrix} + C_2 e^{2t}\begin{pmatrix}0\\1\end{pmatrix} + e^{-t}\begin{pmatrix}4\\3\end{pmatrix}$

(d) $x(t) = C_1 e^{t}\begin{pmatrix}1\\0\end{pmatrix} + C_2 e^{2t}\begin{pmatrix}0\\1\end{pmatrix} + e^{-t}\begin{pmatrix}1\\1\end{pmatrix}$

(e) $x(t) = C_1 e^{t}\begin{pmatrix}1\\0\end{pmatrix} + C_2 e^{2t}\begin{pmatrix}0\\1\end{pmatrix} + e^{-t}\begin{pmatrix}2\\1\end{pmatrix}$

$r = 1, 2$

$(1-r)(2-r) = 0$

$2 - 2r - r + r^2 = 0$

$r^2 - 3r + 2 = 0$

$(r-2)(r-1) = 0$

$r = 1, 2$

$X = c_1 K^{(1)} e^{t} + c_2 K^{(1)} e^{2t}$

$\begin{pmatrix} 1-r & 0 \\ 0 & 2-r \end{pmatrix} X = -e^{-2t}\begin{pmatrix}-3\\-4\end{pmatrix}$

k_1 = any number

$k_1 = 0$

(4) $f(t) = 1 + u_2(t)(t-2) + u_4(t)(2-t) + u_6(t)t^2$ The values of $f(3), f(5)$ and $f(7)$ are

(a) 10, 26, 50.
(b) 1, -3, 49.
(c) 2, 1, 49.
(d) 2, -3, 50.
(e) 2, 1, 50.

$u_2(t) = \begin{cases} 0 & t \le 2 \\ 1 & t \ge 2 \end{cases}$

$(t-2)u_2(t) = \begin{cases} 0 & t < 2 \\ t-2 & t \ge 2 \end{cases}$

$(2-t)u_4(t) = \begin{cases} 0 & t < 4 \\ 2-t & t \ge 4 \end{cases}$

$u_6(t)t^2 = \begin{cases} 0 & t < 6 \\ t^2 & t \ge 6 \end{cases}$

$f(3) = 1 + 1 + 0 + 0 =$

$f(5) = 1 + 3 - 3 + 0 =$

$f(7) = 1 + 5 - 5 + 49 =$

$\Rightarrow$ Q1

(2) ~~$x = 1$~~

indicial eq.

$$r(r-1) + \frac{1}{2}r = 0$$

$\Rightarrow$ sln. $\sum_{n=0} a_n (x-1)^{n+r}$